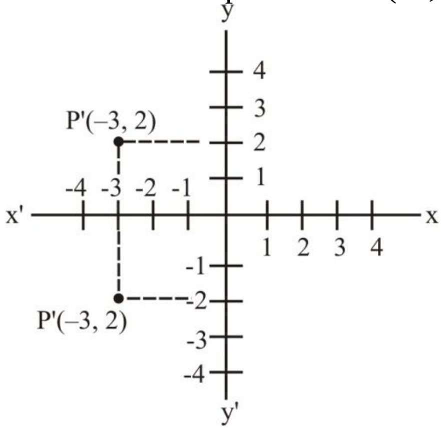
SOLUTION

Math Test

Section – A

Q1.

(i) The reflection of point P is P' (–3, 2) in the x-axis is



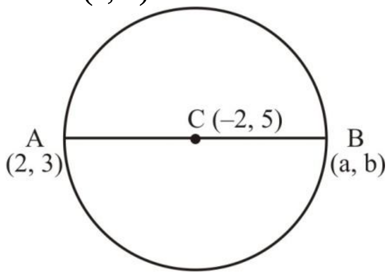
Therefore point P is (–3, –2).

(ii) Let AB is diameter

A(2, 3)

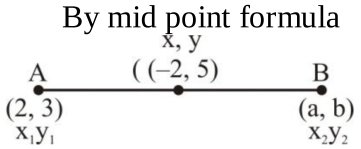
Centre C (–2, 5)

Let B (a, b)



∴ C is centre of AB

By mid point formula



By mid point formula:

$x = \dfrac{x_1 + x_2}{2}$ $y = \dfrac{y_1 + y_2}{2}$

$-2 = \dfrac{2 + a}{2}$ $5 = \dfrac{3 + b}{2}$

–4 = 2 + a 10 – 3 = b

a = –6 b = 7

Therefore coordinate of B(–6, 7)

(iii) Given that: $\theta = 45$ and $C = -3$

$m = \tan 45^\circ = 1$

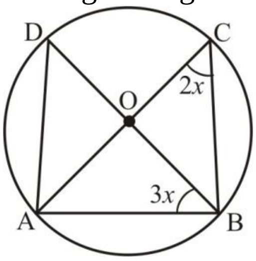
Equation of slope intercepts form $y = mx + c$

$y = 1 \times x + (-3)$

$y = x - 3$

$x - y - 3 = 0$

(iv) In the given figure 'O' is centre of circle



$\angle BAC = 90^\circ$

(angle in a semicircle is 90º)

$\angle ADB = \angle ACB$ (= Angle in same segment are equal)

$\angle ADB = 2x$

In Δ ADB,

$2x + 3x + 90 = 180^\circ$

$5x + 90 = 180$

$5x = 180 - 90$

$5x = 90$

$x = 90/5$

$x = 18^\circ$

(v) Given that

Radius = $\frac{r}{2}$

Slant height = 2ℓ

The total surface area of cone is :

$\pi r(\ell + r)$

$= \pi \frac{r}{2}\left(2\ell + \frac{r}{2}\right)$

$= \pi \frac{r}{\cancel{2}} \cancel{2}\left(\ell + \frac{r}{4}\right)$

$= \pi r\left(\ell + \frac{r}{4}\right)$

(vi) $\tan^2\theta - \sin^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \qquad (\because \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta})$

$\sin^2\theta\left(\frac{1}{\cos\theta} - 1\right)$

$\sin^2\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right) \qquad (\because 1-\cos^2\theta = \sin^2\theta)$

$= \frac{\sin^2\theta}{\cos^2\theta}(\sin^2\theta)$

$= \tan^2\theta \sin^2\theta$

(vii) If $\sec\theta + \tan\theta = k$, then the $\sec\theta - \tan\theta$ = ?

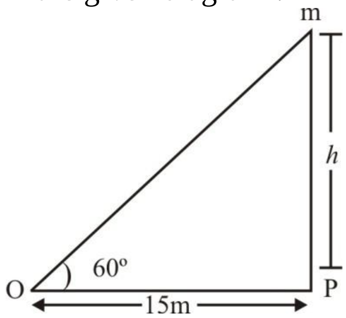
$\because \sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$k(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \frac{1}{k}$

(viii) In the given diagram :



$\tan 60^\circ = \frac{PM}{OP}$

$\sqrt{3} = \frac{h}{15}$

$h = 15\sqrt{3}$

(ix) In the given diagram the maximum frequency class is (40 – 50), so the mode from the given option is 40 which comes between this range.

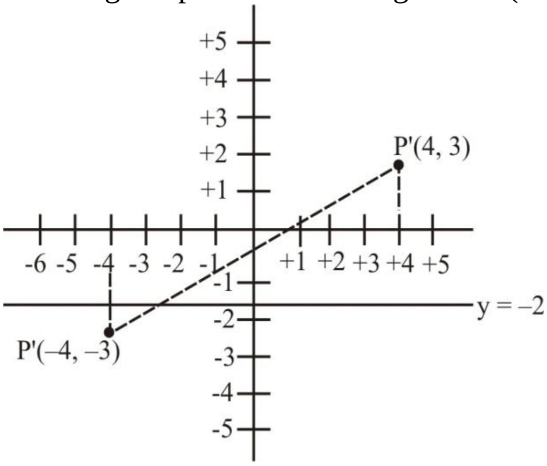
(x) Lakshmi tossed two coins three time so the possible outcomes are 4
{HH, HT, TH, TT}
The favourable event atmost one head {HT, TH, TT} = 3

$\because P(E) = \frac{3}{4}$

Section - B

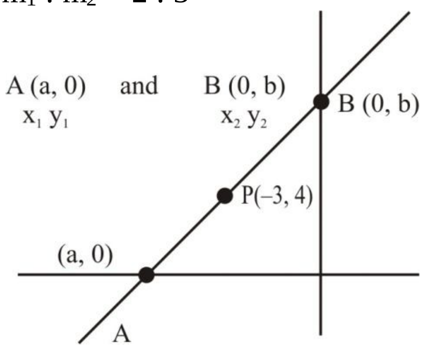
Q2.

(i) (a) The image of point P in the origin is P[1] (–4, –3). So the coordinates of P is (4, 3).



(b) Image of point P in the line y = –2 which is parallel to x-axis is :
P'' (4, –3+2a)
P'' (4, –3+2 (–2)
P'' (4, –3–4)
P'' (4, –7)

(ii) Let coordinate of A (a, 0)
and coordinate B (0, 6)
$m_1 : m_2 = 2 : 3$



By Section Formula :

$$x = \frac{m_1x_2 + m_2y_2}{m_1 + m_2} \qquad y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$-3 = \frac{2\times 0 + 3\times 4}{2+3} \qquad 4 = \frac{2\times b + 3(0)}{2+3}$$

$$-3 = \frac{3a}{5} \qquad 4 = \frac{2b}{5}$$

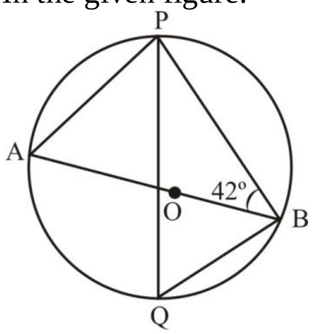
$$-15 = 3a \qquad 20 = 2b$$

$a = \frac{-15}{3}$ $b = \frac{20}{2}$

$a = -5$ $b = 10$

Therefore coordinates of A(–5, 0) and B is (0, 10).

(iii) In the given figure:



O is centre of circle

$\angle PBA = 42^\circ$

and $\angle APB = 90^\circ$

($\because$ angle in a semicircle is 90º)

In ΔAPB

$42^\circ + 90 + \angle PAB = 180^\circ$

$132 + \angle PAB = 180$

$\angle PAB = 180 - 132$

$\angle PAB = 48^\circ$

$\angle BQP = \angle PAB$ (Angle in a same segment)

Therefore $\angle PBQ = 48^\circ$

(iv) Let radius and height of cylinder is 2x and 7x.

Volume of cylinder = $\pi r^{2}h$

$704 = \frac{22}{7} \times (2x)^2 (7x)$

$704 = \frac{22}{\cancel{7}} \times 4x^2 * \cancel{7}x$

$704 = 4 \times 22\ x^3$

$\frac{\cancel{704}}{\cancel{22} \times \cancel{4}} = x^3$

$8 = x^3$

$\therefore\ x = 2$

So Radius = 2x

= 2 × 2

= 4 cm

height = 7x

= 7 × 2

= 14 cm.

Total surface area of cylinder

$= \quad 2\pi r(h+r)$

$= \quad 2\times\frac{22}{7}\times 4(14+4)$

$= \quad \frac{44}{7}\times 4(18)$

$= \quad \frac{3168}{7}$

$= \quad 452.57 \text{ cm}^2$

Q3.

(i) LHS. $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$\frac{\sin\theta(1-2\sin^2\theta)}{\cos(2\cos^2\theta - 1)}$

$= \frac{\sin\theta(1-2(1-\cot\theta)}{\cos(2\cos^2\theta-1)}$

$= \frac{\sin\theta(1-2+2\cos^2\theta)}{\cos\theta(2\cos^2\theta-1)}$

$= \frac{\sin\theta(\cancel{2\cos^2\theta} - 1)}{\cos\theta(2\cos^2\theta-1)}$

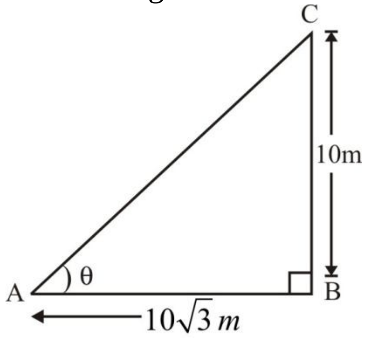
$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta \quad$ (R.H.S.)

(ii) height = 10 m

length of shadow $10\sqrt{3}\ m$

Let 'θ' is angle of elevation



$\tan\theta = \frac{AC}{AB}$

$\tan\theta = \frac{10}{10\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \tan 30°$
$\theta = 30°$
Therefore angle of elevation of sun is 30°.

(iii) For the following distribution table mean is 6.

Variate (x_i)	f_i	$f_i x_i$
2	3	6
4	2	8
6	3	18
10	1	10
p + 5	2	2p + 10
Total	$\Sigma f_i = 11$	$52 + 2p = \Sigma f_i x_i$

$$\text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$6 = \frac{52+2p}{11} \quad \Rightarrow 66 = 52 + 2p$$

$66 - 52 = 2p$
$14 = 2p$

$$p = \frac{14}{2}$$

$p = 7$

(iv) Total triangle = 8 in which 3 blue triangle and 5 red triangle
Total square = 10 in which Blue square = 6 and Red square = 4
Total piece = 10 + 8 = 18
If one piece is lost randomly the probability of a triangle

a) $$P(\varepsilon) = \frac{8}{18}$$

$$= \frac{4}{9}$$

b) A square of blue colour

$$P(\varepsilon) = \frac{6}{18}$$

$$P(\varepsilon) = \frac{1}{3}$$

c) A triangle of red colour :

$$P(\varepsilon) = \frac{5}{18}$$

Q4.

(i) $$\text{mean} = \frac{1+7+5+3+4+4}{6} \quad (\text{mean} = m)$$

$$m = \frac{24}{6}$$

$m = 4$

and mean of 3, 2, 4, 2, 3, 3, P is $(m-1)$

mean = $\dfrac{3+2+4+2+3+3+P}{7}$

$(m-1) = \dfrac{17+P}{7}$

$4-1 = \dfrac{17+P}{7}$

$3 = \dfrac{17+P}{7}$

21 = 17 + P

21 – 17 = P

4 = P

For median : arrange in ascending order

2, 2, 3, 3, 3, 4, 4

$n = 7$ (odd)

median = $\left(\dfrac{n+1}{2}\right)^{th}$ term

median = $\left(\dfrac{7+1}{2}\right)^{th}$

q = $\left(\dfrac{8}{2}\right)^{th}$

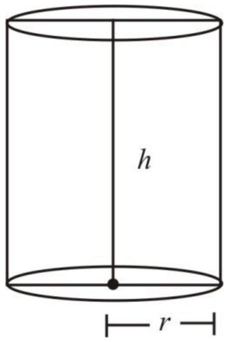
q = 4^{th} term

q = 3

Therefore, P = 4 and q = 3

(ii) Let radius = r, and h = 25 cm (Given)

(a) The circumference of base is 132.



$2\pi r = 132$

$2\times\dfrac{22}{7}\times r = 132$

$r = \dfrac{132\times 7}{22\times 2}$

r = 21 cm.

(b) Volume of cylinder = $\pi r^2 h$

= $\frac{22}{7} \times (21)^2 \times 25$

= $\frac{22}{7} \times 21 \times 21 \times 25$

= $22 \times 3 \times 21 \times 5$

= 34650 cm^3

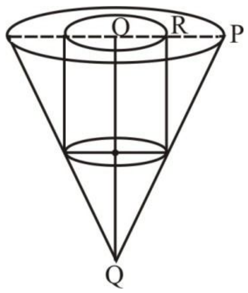
(iii) For cone:
Radius OP = 7 cm
Height OQ = 12 cm
For cylinder
Radius = half of radius of cone

= $\frac{1}{2} \times 7$

$R_{cylinder} = \frac{7}{2}$

and height of cylinder = $\frac{1}{2} \times h$

= 6 cm



If cylinder of radius $\frac{7}{2}$ and height 6 cm. is drilled out from cone, remaining volume

= Volume of cone – volume of cylinder

= $\frac{1}{3}\pi r_1^2 h_1 - \pi r_2^2 h_2$

= $\frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 12 - \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 6$

= $\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12 - \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 6$

= $22 \times 7 \times 4 - \frac{22 \times 7 \times 7}{2}$

= $616 - 11 \times 7 \times 3$

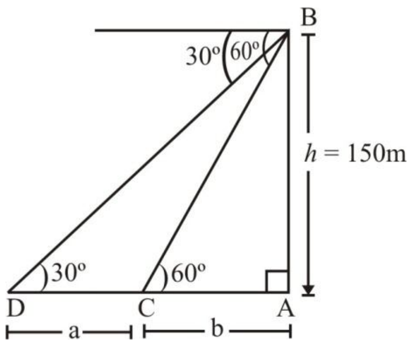
= $616 - 231$

= 385 cm^3

(iv) We have :

$$\frac{4}{3}(\sec^2 59 - \cot^2 31) - \frac{2}{3}\sin 90^o + 3\tan^2 56.\tan^2 34 = \frac{x}{3}$$
$$\frac{4}{3}(\sec^2 59 - \cot^2(90-59)) - \frac{2}{3}\times 1 + 3\tan^2 56.\tan^2(90-56) = \frac{x}{3}$$
$$\frac{4}{3}(\sec^2 59 - \tan^2 59) - \frac{2}{3} + 3\tan^2 56\cot^2 56 = \frac{x}{3}$$
$$\frac{4}{3}(1) - \frac{2}{3} + 3 = \frac{x}{3}$$
$$\frac{4}{3} - \frac{2}{3} + 3 = \frac{x}{3}$$
$$\frac{4-2}{3} + 3 = \frac{x}{3}$$
$$\frac{2}{3} + 3 = \frac{x}{3}$$
$$\frac{2+9}{3} = \frac{x}{3}$$
$$\frac{11}{3} = \frac{x}{3}$$
x = 11

Q5.



(i)

Let AB is a cliff of h = 150m.
And two boats are at C and D and angle of depression are 60° and 30° respectively.
Let 'a' is distance between two boats
In ΔABC

$$\tan 60 = \frac{AB}{AC}$$
$$\sqrt{3} = \frac{150}{b}$$
$$b = \frac{150}{\sqrt{3}} = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{7}}$$
$$= \frac{150\sqrt{3}}{3}$$

$b = 150\sqrt{3}$

In Δ DAB.

$\tan 30^\circ = \frac{AB}{AD}$

$\frac{1}{\sqrt{3}} = \frac{150}{AC + CD}$

$\frac{1}{\sqrt{3}} = \frac{150}{b + a}$

$a + b = 150\sqrt{3}$

$a = 150\sqrt{3} - 50\sqrt{3}$

$a = 100\sqrt{3}$

a = 100 × 1.732

a = 173.2m

Therefore the distance between two boats are 173.2m

(ii) Total marbles in a jar = 24

Let 'x' are green and (24 – x) are blue.

Probability of green = $\frac{x}{24}$

and P(G) = $\frac{2}{3}$

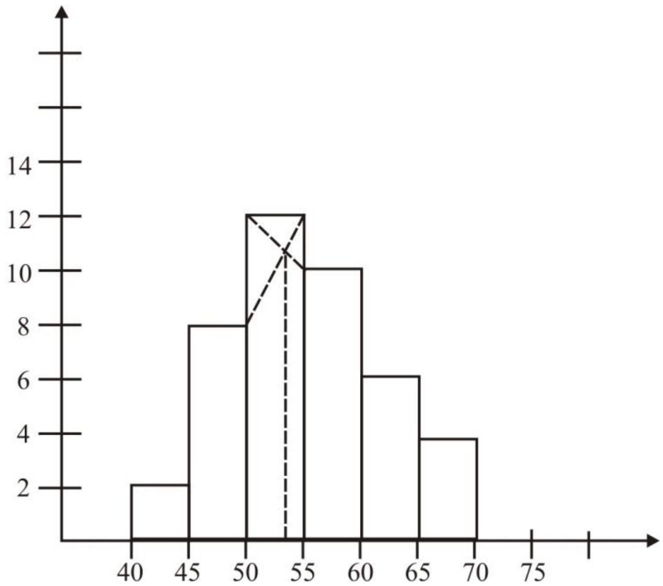
$\frac{2}{3} = \frac{x}{24}$

$x = \frac{24 \times 2}{3}$

x = 16

So the number of blue balls = 24 – 16
= 8

(iii) Histogram



mode = 57

(iv) If $\tan\theta + \sec\theta = 1$ ....(1)

$\because \quad \sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\ell(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \frac{1}{\ell}$ ....(2)

Add (1) + (2)

$\sec\theta + \tan\theta = \ell$

$\sec\theta - \tan\theta = \frac{1}{\ell}$

$2\sec\theta = \ell + \frac{1}{\ell}$

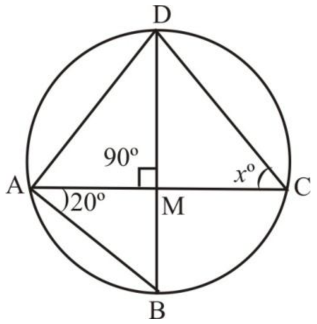
$2\sec\theta = \frac{1+\ell^2}{\ell}$

$\sec\theta = \frac{1+\ell^2}{\ell * 2}$ Hence prove

Q6.

(i) In the given figure

$\angle BDC = \angle CDB = 20^\circ$ (angle in same segment are equal)



and ∠DMC = 90°

In Δ DMC,

20 + 90 + x = 180°

110 + x = 180

x = 180 – 110

x = 70°

(ii) Given that (a, 2a) line on the line $\frac{y}{2}$ = 3x – 6, which satisfies the given equator.

$\frac{y}{2} = 3x - 6$

$\frac{2a}{2} = 3 \times a - 6$

a = 3a – 6

6 = 3a – a

6 = 2a

$a = \frac{6}{2} = 3$

Therefore the value of a is 3.

(iii) The equation is parallel to the line joining the point

$A(\underset{x_1}{7},\underset{y_1}{-1})$ and $B(\underset{x_2}{0},\underset{y_2}{3})$

slope of line $AB = \frac{y_2 - y_1}{x_2 - x_4}$

$m = \frac{3-(-1)}{0-7}$

$m = \frac{3+1}{-7}$

$m = -\frac{4}{7}$

and if two lines are parallel then their slope are also equal so the slope of required line is also $\frac{-3}{7}$

So the equation of line passing through

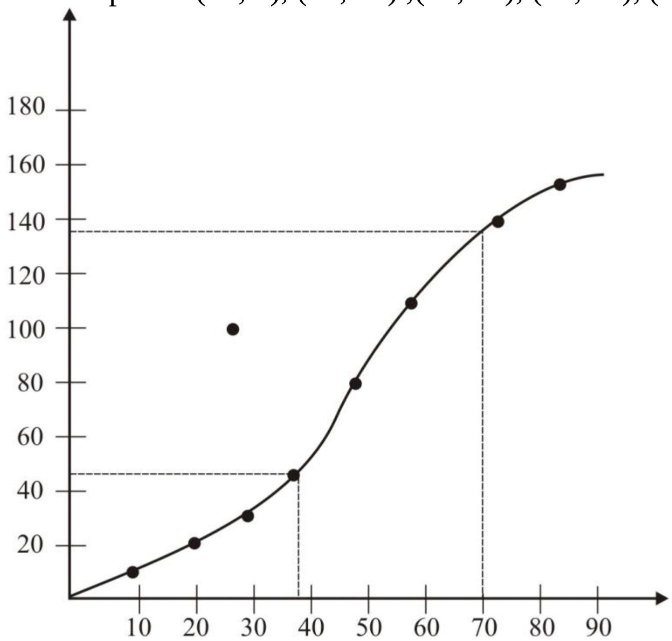
P(–5, 1) having slope $\frac{-3}{7}$ is

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{-4}{7}(x - (-5))$$
$$y - 1 = \frac{-4}{7}(x + 5)$$
$$7(y - 1) = -4x - 20$$
$$7y - 7 = -4x - 20$$
$$7y - 7 = -4x - 20$$
$$4x - 7y - 7 + 20 = 0$$
$$4x - 7y + 13 = 1$$

(iv)

CI	*fi*	*C.F*
0-10	5	5
10-20	10	15
20-30	11	26
30-40	20	46
40-50	27	73
50-60	38	111
60-70	40	151
70-80	29	180

Taking 1cm = 10 marks – x-axis and
1cm = 20 students on y-axis
Plot the points (10, 5), (20, 15) ,(30, 26), (40, 46), (50, 73), (60, 111), (70, 151), (80, 180).



Here $n = 180$

(i) Lower quartile $= \frac{n}{4}$

$= \frac{180}{4}$

$= 45$

Through B, draw a horizontal line to meet the ogive at Q. Through Q, draw a vertical line to meet the x-axis at N. at 39

so lower quartile = 39 marks.

(ii) Upper quartile $= \frac{3n}{4}$

$= \frac{3 \times 45}{4}$

$= 3 \times 45$

$= 135$

Through A draw a line to meet the ogive at P, through P draw a line vertical to met the x-axis at M at 64.

So upper quartile is = 64